(**WORK FIVE PROBLEMS: TWENTY POINTS EACH; EXTRA CREDIT TWO POINTS EACH**)

1.) a.) Given $G(\vec{r})$ is a scalar valued function with continuous derivatives through second order. Use the Cartesian representation to show that: $\vec{\nabla} \times \vec{\nabla} G = 0$. This result is summarized in the operator relation $\vec{\nabla} \times \vec{\nabla} = 0$.

b.) Really strange weather led to a wind velocity pattern over the surface of the earth that had the form $\vec{v}(r,\theta,\phi) = (3\,{}^{m}\!/_{s})\left[-\sin\theta\ \sin\phi\ \hat{\theta} + 2\sin\theta\ \hat{\phi}\right]$. The pattern being independent of r in the region of interest. Compute $\vec{\nabla} \times \vec{v}$.

2.) a.) A rotation of coordinates by θ about the x-axis is represented as

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

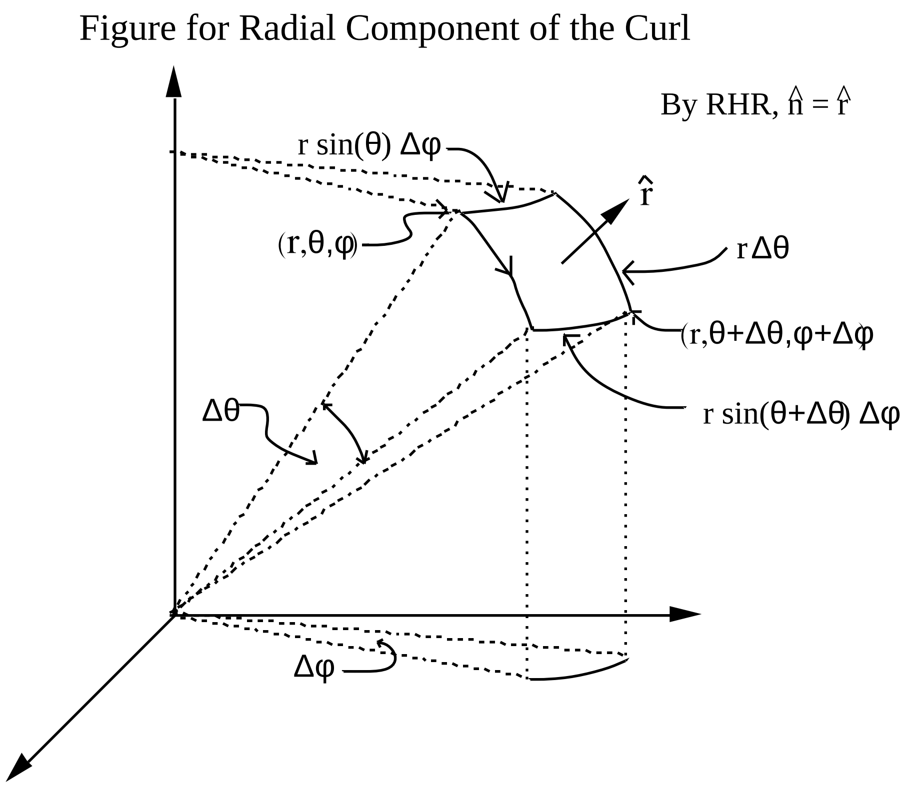
, and a rotation of coordinates by φ about the z-axis is represented as

$$\begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Find the matrix that represents a rotation of coordinates by 45 degrees about ***the z-axis*** followed by 90 degrees about the then ***x-axis***.

b.) Prior to the rotation of coordinates, a position vector was reported to have components (1*m*, 1*m*, 1*m*). What is its component representation in the new reference frame after the rotations of part a.) ?

3.)



Compute:

$\int \vec{v} \cdot d\vec{r}$ for the path shown with θ and $\varphi = \pi/4$ and

$\Delta\theta = \Delta\varphi = \pi/12$

$\pi/4 < \theta < \pi/6$

$\pi/4 < \varphi < \pi/6$

$\vec{v} = \frac{\theta}{\sin\theta}\hat{\phi} - \phi\,\hat{\theta}$

ANY METHOD !!

4.) Below, $\mathbb{M}$ is represented relative to an initial set of Cartesian axes with the mapping [$x_1 \leftrightarrow x$; $x_2 \leftrightarrow y$;$x_3 \leftrightarrow z$]. Use the method from the rotated ellipse example to find the angle to rotate the axes into the eigendirections. Give the eigenvlaues and eigendirections for the problem. The value that you get for *tan*(θ) may be unexpected. Ask yourself, what angel has this value as its tangent.

Given $\mathbb{M} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, find the eigenvalues and eigenvectors.

May be solved as a classic eigenvlaue problem for 75% credit.

5.) a.) Many quantities of physical interest are represented by real symmetric matrices. What does it mean to say a matrix is real symmetric ?

b.) Passive rotations of reference frames are represented by orthogonal matrices. What does it mean to say that a matrix is orthogonal ?

c.) The matrix λ is a 3 x 3 real orthogonal matrix. Use the properties of orthogonal matrices to compute $\sum_{i=1}^{3} \lambda_{1i}\ \lambda_{3i}$. Explain !

d.) Prove one of the following:

i.) The determinant of an orthogonal matrix is ± 1.

ii.) The eigenvectors of a real symmetric matrix corresponding to distinct eigenvalues are orthogonal.

X-tra Credit

(***2 pts***.) Define and contrast *active* and *passive* transformations.

(***2 pts***.) Electromagnetic theory states that $\vec{B}(\vec{r}) = \vec{\ } \times \vec{A}(\vec{r})$. Given the magnetic field described in cylindrical coordinates as $B_0\,({}^{r}\!/\!{}_{R})\hat{\theta}$ and $B_0\,({}^{R}\!/\!{}_{r})\hat{\theta}$ for r less than and greater than R respectively, find $\vec{A}(\vec{r})$ for all r given that $\vec{A}(0) = 0$.

(***2 pts***.) Develop the curl in cylindrical coordinates as Grad-cross-Vector. Note that the differential operators in the gradient operator must be allowed to operate on the coordinate directions in the representation of the vector prior to computing the cross (exterior) products.

(***2 pts***.) Beginning with an excellent figure, develop the contributions to the divergence in spherical coordinates associated with the faces of the coordinate 'cube' with normal directions $\hat{r}$ and $-\hat{r}$.