4 problems + 4 extra credit of which no more than 3 can be solved for credit
25 points per problem divided equally among the parts.
2 points per extra credit problem – not worth it ! *max extra credit + 6%*

1.) The classic eigenvalue problem: $\mathbb{M}\,\vec{v} = \lambda\,\mathbb{1}\,\vec{v} = \lambda\,\vec{v}$

Given $\mathbb{M} = \begin{bmatrix} 5 & 1 & 0 \\ 1 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, find the eigenvalues and eigenvectors.

2.) a.) Explain the procedure to construct the transformation matrix that diagonalizes $\mathbb{M}$ using the eigenvectors found while solving problem #1.

Give the transformation matrix λ which could be used to diagonalize $\mathbb{M}$.

b.) $|\ \mathbb{A}\ | = 4$ $|\ \mathbb{B}\ | = 2$ Give the values of each determinant below based on the values for $|\ \mathbb{A}\ |$ and $|\ \mathbb{B}\ |$.

b_1.) $|\ \mathbb{A}\ \mathbb{B}\ | =$

b_2.) $|\ \mathbb{A}\ \mathbb{B}^{-1}\ |$

b_3.) If $\mathbb{B}$ is a 2 x 2 matrix, $|\ 3\ \mathbb{B}\ | =$

b_4.) $|\ \mathbb{A}^{t}\ | =$

3.) The matrix representation for a rotation by an angle φ about the z axis is:

$$\lambda_{\mathbf{z}}(\phi) = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ and by θ about the x axis is: $$\lambda_{\mathbf{x}}(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}.$$

a.) Give the matrix that represents a rotation of $\pi/2$ about the z-axis followed by a rotation of $\pi/2$ about the then x-axis.

b.) A vector has a component representation $\vec{V} = 2\,m\,\hat{i} + 11\,m\,\hat{j} + 7\,m\,\hat{k}$ in an original frame. Give the component representation of this vector in a frame rotated by $\varphi = \cos^{-1}(0.6) = +\,53.13^0$ about the z axis relative to the original frame.

c.) A tensor $\mathbb{A}$ has the representation $\tilde{\tilde{A}} = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 6 & 1 \\ 2 & 1 & 4 \end{bmatrix}$ in the original frame. Using the transformation matrix found in part b, write down the explicit matrix multiplication that would give $\tilde{\tilde{A}}'$, the form of the matrix as expressed with respect to the rotated frame. DO NOT MULTIPLY !!

4.) Express the following set of linear algebraic equations in matrix form and then solve them using Cramer's rule.

$$\begin{aligned} x + y &= 4 \\ x - y &= 2 \\ x + y + z &= 2 \end{aligned}$$

a.) The matrix form of the equations is:

b.) The complete Cramer's rule solution is:

$$\begin{aligned} x + 2y - z &= 0 \\ x - y + z &= 0 \\ 3x + z &= 0 \end{aligned}$$

c.) This system of equations might appear to have only the trivial solution: all unknowns equal zero. Under what conditions can a set of equations equal to a constant vector of zeroes have a non-trivial solution ?

each extra credit exercise is only + 2% *max extra credit + 6%*

X1.) Use the transformation matrix found in #2 to diagonalize the matrix $\mathbb{M}$ studied in problem #1.

X2.) Show that the multiplication of *n x n* matrices by *n x n* matrices does not commute in all cases.

X3.) Show that a similarity transformation of a real symmetric matrix with a real orthogonal matrix and its inverse results in a real symmetric matrix.

X4.) Show that $\lambda_z(\phi) = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is the correct form for a matrix representing a transformation to a coordinate frame with the same origin and z-axis that is rotated by an angle φ about the z-axis relative to the original frame. A good method is to develop the transformation for the components of $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$

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