5 problems + 3 extra credit
20 points per problem divided equally among the parts.
2 points per extra credit problem – not worth it !

1.) a.) Give the expression for the line element $d\vec{r}$ in Cartesian, cylindrical and spherical coordinates.

b.) Sketch a coordinate 'cube' (a small volume element with pieces of the line element as three of its edges) in ***cylindrical*** coordinates that has one corner at $(2, {}^{\pi}/_{4}, 1)$. Highlight the three edges that correspond to components of the line element.

c.) Give the expressions for the ***spherical*** coordinate directions $\hat{r}$ and $\hat{\theta}$ in terms of $\hat{i}$, $\hat{j}$ and $\hat{k}$.

d.) Give the ***spherical*** coordinate representation of the volume element dV.

 Name: ____________________________

2.) Complex numbers

a.) $z = 3 + i\ 4;$ $z^* =$ __________________

b.) $z = 3 + i\ 4;$ $z^{-1} =$ ___________________

c.) Simplify $\frac{1+i}{1-i} =$ ________________________

d.) Represent $z = 3 + i\ 4$ *in polar form*. = _________________

e.) Represent $z^{-1} = [3 + i\ 4]^{-1}$ *in polar form*. = _________________

f.) Compute any square root of $3 + i\,4$. $\sqrt{3+i4} =$ ______________

3.) An even function of time with a period of 10 seconds and average value zero is to be expanded in a Fourier series.

a.) What are the allowed frequencies for this Fourier expansion ?

b.) Given the expansion $f(t) = c_{f0} + \sum_{m=1}^{\infty} a_{fm} \cos[\omega_m t] \quad + \sum_{m=1}^{\infty} b_{fm} \sin[\omega_m t]$, what does the coefficient c_{f0} represent ?

c.) Given the character of the function to be expanded, which of the expansion coefficients are expected to be zero.

d.) Use the orthogonality relations to generate an expression for the value of the expansion coefficient a_{f4}.

4.) a.) A vector space is a set of elements for which two operations are defined. What are they ? Describe them very briefly.

b.) Closure is required for both operations. Explain what is required by closure.

c.) Choose one of the two operations. In addition to closure, it satisfies four axioms. List them. At minimum provide the equation form of each axiom. If time permits briefly explain them.

5.) a.) A certain set is a basis set for the vector space $\mathbb{V}$. What does that mean ?

b.) The Gram-Schmidt procedure begins by removing the components of a vector along the vectors previously added to the basis.

$$\left|E_j\right\rangle = \left|B_j\right\rangle - \sum_{i<j} \frac{\left\langle b_i \middle| B_j \right\rangle \left|b_i\right\rangle}{\left\langle b_i \middle| b_i \right\rangle}$$

What is the additional step required to find the final form for an addition to the ortho-normal basis ?

c.) Use the Gram-Schmidt procedure to create an ortho-normal basis from the spanning set $\left\{\hat{i} + \hat{j};\, 3\hat{i} + \hat{j}; 2\,\hat{j} + \hat{k}\right\}$. What is the dimension of the space spanned by the set ?

X1.) A set of n vectors $\{|V_1\rangle, |V_2\rangle, \ldots\ldots, |V_n\rangle\}$ that are elements of the vector space $\mathbb{V}$ are found to be linearly dependent, and it is found that the zero vector is not a member of this set. Starting with the definition for linear dependence, prove that at least one vector in the set other than $|V_1\rangle$, the first one listed, can be represented as a linear combination of the other vectors in the set.

Remove the vector that was a linear combination of the other $n-1$. Show that the span of the remaining n-1 vectors is the same as $SPAN(\{|V_1\rangle, |V_2\rangle, \ldots\ldots, |V_n\rangle\})$, the span of the original set of n vectors.

X2.) Evaluate $1-\cos(m\frac{\pi}{2})$ for all non-negative integers m.

X3.) Compute $\dot{\vec{r}}$ and $\ddot{\vec{r}}$ in polar coordinates (cylindrical omitting z).

THINGS TO USE

The Orthogonality Relations for the Fourier Expansion Set

$$\frac{1}{T}\int_{-T/2}^{T/2}[1]\ [1]\ dt = 1; \qquad \frac{1}{T}\int_{-T/2}^{T/2}[1]\sin[\omega_m t]\ dt = 0_{pm}; \qquad \frac{1}{T}\int_{-T/2}^{T/2}[1]\cos[\omega_m t]\ dt = 0$$

$$\frac{1}{T}\int_{-T/2}^{T/2}\sin[\omega_p t]\ \cos[\omega_m t]\ dt = 0; \qquad \frac{1}{T}\int_{-T/2}^{T/2}\sin[\omega_p t]\ \sin[\omega_m t]\ dt = \tfrac{1}{2}\,\delta_{pm}; \qquad \frac{1}{T}\int_{-T/2}^{T/2}\cos[\omega_p t]\ \cos[\omega_m t]\ dt = \tfrac{1}{2}\,\delta_{pm}$$

$$\sin[m\pi] = \begin{cases} 0 & for\ m\ even;\ m = 2p \\ 0 & for\ m\ odd;\ m = 2p+1 \end{cases} \qquad \cos[m\pi] = (-1)^m = \begin{cases} 1 & for\ m\ even;\ m = 2p \\ -1 & for\ m\ odd;\ m = 2p+1 \end{cases}$$

$$\sin\left[m\left(\frac{\pi}{2}\right)\right] = \begin{cases} 0 & for\ m = 4p = 4,8,12,\ldots \\ 1 & for\ m = 4p+1 = 1,5,9,\ldots \\ 0 & for\ m = 4p+2 = 2,6,10,\ldots \\ -1 & for\ m = 4p+3 = 3,7,11,\ldots \end{cases} \qquad \cos\left[m\left(\frac{\pi}{2}\right)\right] = \begin{cases} 1 & for\ m = 4p = 4,8,12,\ldots \\ 0 & for\ m = 4p+1 = 1,5,9,\ldots \\ -1 & for\ m = 4p+2 = 2,6,10,\ldots \\ 0 & for\ m = 4p+3 = 3,7,11,\ldots \end{cases}$$

$$\sin\left[m\left(\frac{\pi}{4}\right)\right]=\begin{cases} 0 & \text{for } m=8p \\ 1/\sqrt{2} & \text{for } m=8p+1 \\ 1 & \text{for } m=8p+2 \\ 1/\sqrt{2} & \text{for } m=8p+3 \\ 0 & \text{for } m=8p+4 \\ -1/\sqrt{2} & \text{for } m=8p+5 \\ -1 & \text{for } m=8p+6 \\ -1/\sqrt{2} & \text{for } m=8p+7 \end{cases}$$

$$\cos\left[m\left(\frac{\pi}{4}\right)\right]=\begin{cases} 1 & \text{for } m=8p \\ 1/\sqrt{2} & \text{for } m=8p+1 \\ 0 & \text{for } m=8p+2 \\ -1/\sqrt{2} & \text{for } m=8p+3 \\ -1 & \text{for } m=8p+4 \\ -1/\sqrt{2} & \text{for } m=8p+5 \\ 0 & \text{for } m=8p+6 \\ 1/\sqrt{2} & \text{for } m=8p+7 \end{cases}$$